The meteorology of the cubic Earth (Part 1)

1. Introduction

The Earth looks like a sphere, but exactly speaking, it is a spheroid of a radius of 6,378 km(equator) and 6,357 km(polar) covered by an atmosphere and oceans. The atmospheric pressure above the ground is approximately 1atm wherever of the surface of the Earth. The oceans cover 70% of the Earth's surface and their mean depth is 3.8 km. Of course, they are basic assumptions in meteorology and oceanography.

However, it is necessary to understand the reason that atmosphere environment is in such a condition when the global environment in the future is considered. In other words, a viewpoint that whole global environment as a system is necessary. The knowledge about the real Earth is so empirical that the characteristics of the Earth being sphere are difficult to see.

If the Earth is not a sphere, the global environment will greatly different from real atmospheric and oceanic environment. The characteristics of the Earth being sphere will become clear by comparing the real and imaginary environment.

In this commentary, we consider the phenomena of the atmosphere and oceans when the Earth is cube to clarify the characteristics of the real Earth.

Notice

This article is a commentary text of the movie "If the Earth shape is cube". Numerical values in this article are partly different from numerals used in the movie, because models dealt with are different between the article and the movie. However, the difference is small.

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2. Introduction to the cubic Earth

What kind of the cubic Earth should be studied? Since the cubic Earth is an imaginary planet, arbitrary size of cubic Earth is possible to imagine. Here, let's introduce a cubic Earth which is similar to the real Earth as much as possible.

The circumference of the equator of the real Earth is 40,000 km. Taking this size into consideration, we introduce a cubic Earth with the length of 10,000 km. The volume of this cube is 10^{12}km^3. On the other hand, the radius of the real Earth is approximately 6,400 km, so that its volume is

$$\frac{4}{3}\pi \times \left(6400\,\text{km}\right)^3 = 1.098 \times 10^{12}\,\text{km}^3$$

The volume of the cubic Earth is slightly smaller than the real Earth.

We place the cubic Earth at the location of the real Earth in the solar system. In other words, the cubic Earth rotates around the sun 150,000,000 km apart from the sun with a period of 365 days. The axis makes 23.4 degrees from the normal of the orbit plane in the same way as the real Earth, and spins with a period of approximately 24 hours (exactly speaking, 23 hours 56 minutes).

In the case of the real Earth, rotation axis is the same anywhere, but in the case of the cubic Earth, the global environment is different by a position of the rotation axis.

Figure 1 shows the two cases of symmetric rotation axis. The environment of the cubic Earth greatly changes whether a pole is located in the central square or at the top of the square. We assume here that a pole is located in the central of the square (Figure 1(a)). In addition, we assume that the density inside of the cubic Earth is constant and its mass is same with that of the real Earth.
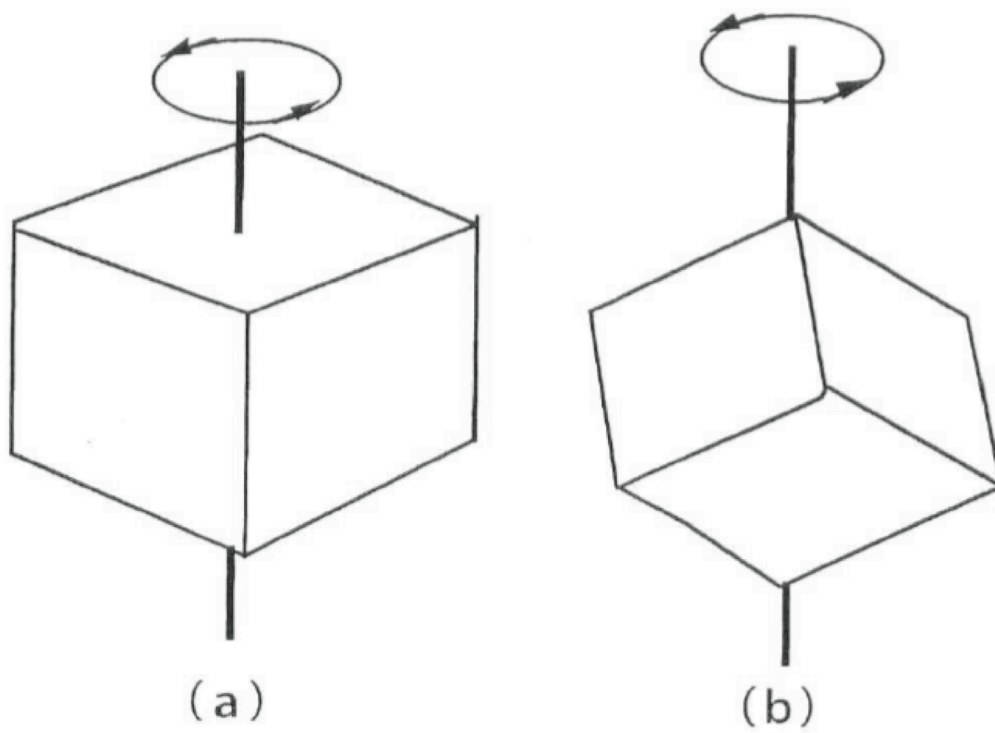
Figure 1: Position of the rotation axis of the cubic Earth (bold vertical line).
 (a) Pole in the center of the square
 (b) Pole at the top of the square

3. Gravitational field of the cubic Earth

3.1 Consideration about the gravitational field

Gravity pulls things toward the center of the Earth (Figure 2). Magnitude of the gravity depends on the distance from the center of the Earth. In the case of the real Earth, the distance from the center of the Earth is constant almost everywhere on the Earth's surface (Figure 2(a)), so the magnitude of the gravitational force is almost same. (More precisely, the force varies slightly depending on latitude, because the Earth is ellipsoid.) Taking the magnitude of gravity on the vertical axis and taking the distance measured along the surface on the horizontal axis, the graph is as shown in Figure 3(a). That is, the value anywhere on the Earth's surface is equal. And the direction of the gravity is directly below to the horizontal plane. So, it is substantially normal to the Earth's surface.

On the other hand, in the case of the cubic Earth, because the distance from the center of the Earth varies depending on the location, the gravity changes as shown in Figure 3(b) (More precise discussion, see Section 3.3). And the direction of gravity is not normal to the ground surface as shown in Figure 2(b). In particular, the corner of the cubic Earth is like a steep mountain. Around the summit of Mount Fuji, the ground surface is inclined about 30 degrees with respect to the horizontal plane. "The Red Fuji" in the prints of Katsushika Hokusai exaggerates the angle of the Mt. Fuji so that the angle near the summit is 53 degrees. In contrast, the angle around the corner of the cubic Earth is 45 degrees. It would be to look like a ridge of the steep mountain.
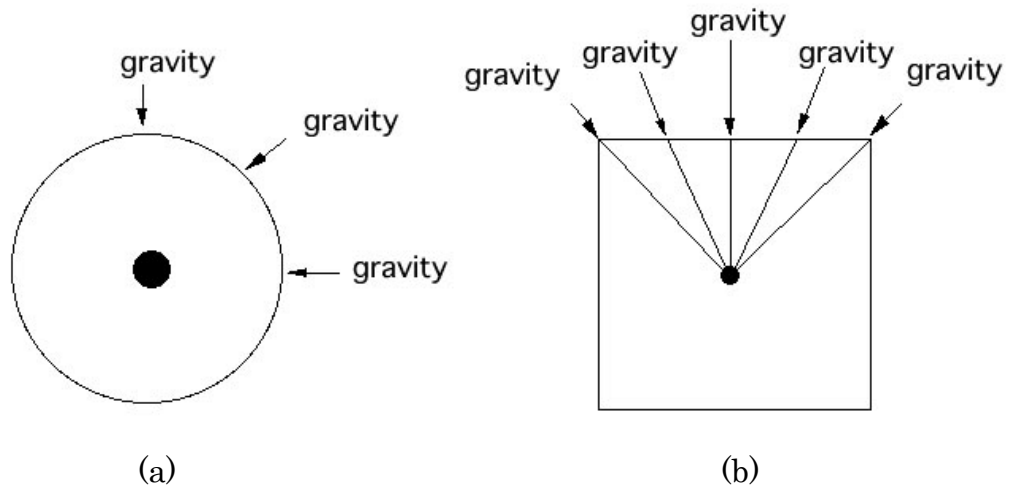
Figure 2: How gravity attracts an object toward the center of the Earth.
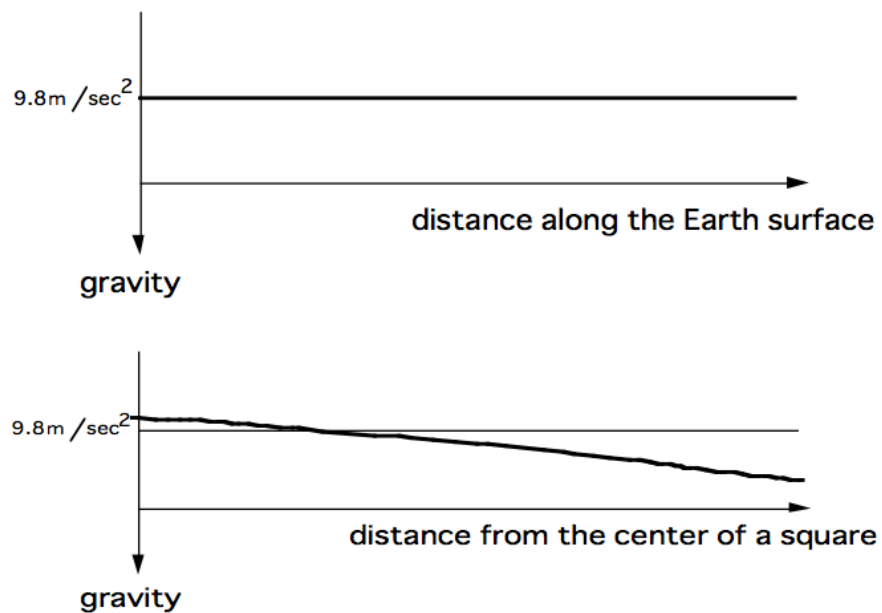 (a) Real Earth.
  (b) Cubic Earth.

Figure 3: Distribution of the magnitude of the gravity at the surface.
  (a) Real Earth.
  (b) Cubic Earth.

3.2 The law of universal gravitation

In general, gravity consists of the centrifugal force due to spinning of the Earth and universal gravitation due to the mass of the Earth.

A universal gravitation is the force of material mass M and mass m at a distance r pulling each other. The force F is

$$F = \gamma \frac{mM}{r^2} \tag{1}$$

The unit of force is Newton [N], where γ is the universal gravitational constant. This law was discovered by English physicist I. Newton in 1665.

Here, m is the mass of the object that are present on the surface of the Earth, and M is the mass of the Earth. In the case that the Earth has spherical symmetry, its universal gravitation is the same as universal gravitation when all mass of the Earth gathers at the centrosphere (Gauss' law). Therefore, equation (1) can be applied to the gravity acting on the object on the Earth, by using the distance r between the object and the center of the Earth.

Gravity is found to be proportional to the mass m of the object. So, putting

$$g = \gamma \frac{M}{r^2} \tag{2}$$

gravity acting on the object is given by mg, where g is called gravitational acceleration. The g at the Earth's surface is a constant that does not depend on m (See Figure 3(a)). The gravitational acceleration at the surface of the Earth, assuming that $r = R_E$ (Earth radius is about 6,400 km), is approximately 9.80 m/s². Going to the sky from the Earth's surface, the gravitational acceleration is reduced because r increases.

As a convention, the distribution of gr is used instead of the distribution of the magnitude of the gravitational acceleration. gr is called the gravitational potential. In the graph of gr, the gradient dgr/dr is the gravitational acceleration.

The law of universal gravitation resembles the electrostatic Coulomb's law in electricity closely. Gravitational acceleration corresponds to the electric field, and gravitational potential corresponds to the electrical potential. An isosurface of the gravitational potential corresponding to the equipotential

surface is called geoid. Form of sea surface which covers the surface of the Earth is a geoid.

 In the real Earth, when you release a heavy object from your hand, the object falls downward. This holds true everywhere on the surface of the Earth. It is because the surface of the Earth is almost a geoid. The magnitude of the gravitational acceleration is also a similar value anywhere on the same sea level. Such property is established because the Earth is almost a sphere. In the case of cubic Earth, direction and magnitude of the gravitational acceleration varies greatly by location on the surface.

3.3 Gravity on the cubic Earth

 Obtaining the gravity at the surface of the cubic Earth is a difficult problem compared with obtaining the gravity of the Earth with a spherical symmetry. Let ρ the density of the Earth and P_G the gravitational potential of a point of the Earth's surface that is made by a small part of the Earth's interior (its volume is $dxdydz$). P_G is given as shown in Figure 4.
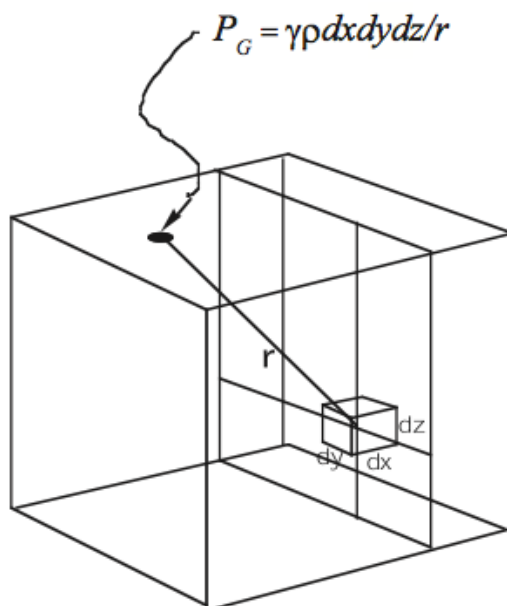
$$P_G = \gamma \frac{\rho dxdydz}{r} \qquad (3)$$

Figure 4: How to determine the gravitational potential of the cubic Earth

To calculate the gravitational potential of the whole cubic Earth, it is necessary to sum up the gravitational potential that is made by a small part of the cubic Earth's interior. Gravitational potential at the center of the square surface of the cubic Earth can be calculated analytically because of the symmetry. Figure 5(a) is a vertical distribution of gravitational potential calculated analytically. Gravitational acceleration at the center of the surface of the cubic Earth is larger than 5% of the real Earth. In addition, the acceleration due to gravity decreases with altitude. Fig.5b is a same graph that is plotted in log-log scale. If the graph plot is in a straight line, it would indicate that the power law of r. Because it is almost linear, it is shown that the law of inverse square is satisfied approximately.

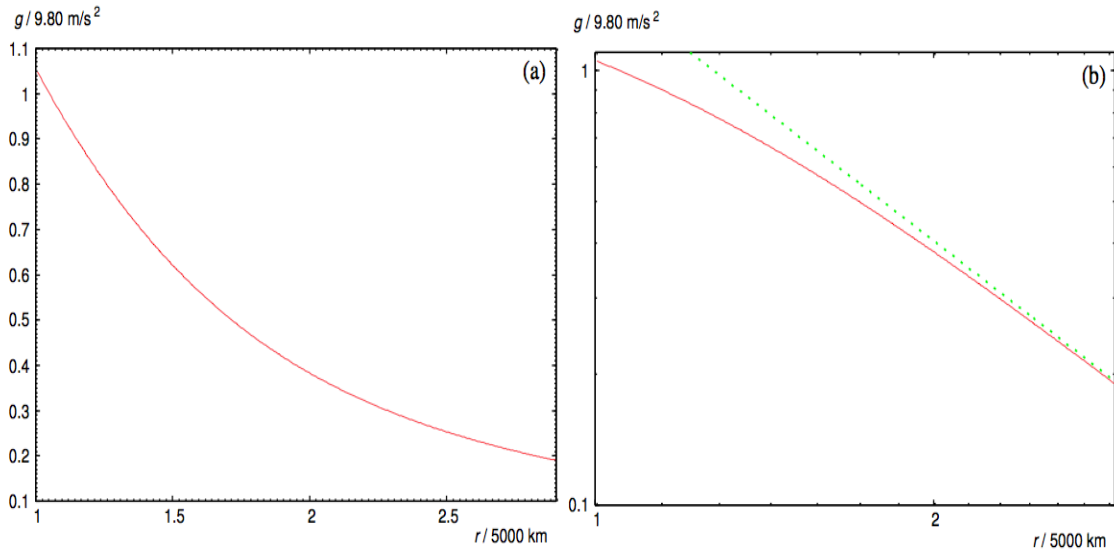Figure 5: Vertical distribution of the gravitational acceleration at the center of the surface of one of the cubic Earth. The vertical axis indicates the gravitational acceleration with the unit of 1 to 9.80 m/s². The horizontal axis indicates the distance from the center of the cube with the unit of 5000 km. Law of inverse square is satisfied approximately.

(a) normal plot.

(b) log-log plot.

Figure 6(a) is the numerical result of the distribution of the gravitational potential at the surface of the cubic Earth. It is shown that the gravitational potential is largest at the center and reduces toward the periphery of the square. This indicates that the direction of gravity is not normal to the square plane. And if the gravitational potential is in the spherical symmetry as the real Earth, gravitational potential lines should be in a circle. But the results that it slightly distorted from a circle. This distortion is the effect of the shape. However, it would be thought approximately as circles. In other words, the distribution of gravity is very similar to the case of the real Earth. Figure 6(b) is a contour line of the gravitational potential in the 1/4 of the cross section obtained by bisecting the middle of the side of the cubic Earth. Also in this figure, the gravitational potential is distributed approximately spherical. So, let the distribution of gravity of the cubic Earth be considered approximately same as the real one.
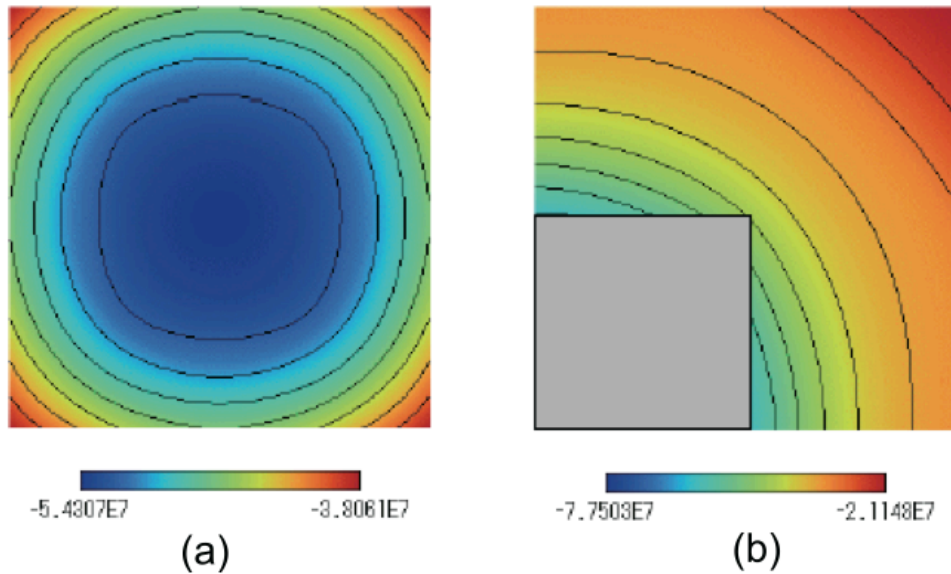

Figure 6: Gravitational potential distribution of the cubic Earth. Numerical unit is J/kg.

 (a) distribution at the square surface.

 (b) distribution in the area of the 1/4 in the cross section bisected so as to pass through the center of the side of the cubic Earth. The gray area represents the cubic Earth's interior.

3.4 Distribution of gravitational potential at the surface of the cubic Earth

  The geoid is a horizontal plane for human. In other words, the surface of the real Earth is spherical, but we feel it as if the horizontal plane surface. Similarly, the horizontal plane is spherical in the case of cubic Earth. Therefore, we feel the surface of the cubic Earth concave. Figure 7 shows the vertical distribution relative to some gravitational potential surface in the two sections of the cubic Earth (Assuming a geoid of the Earth).
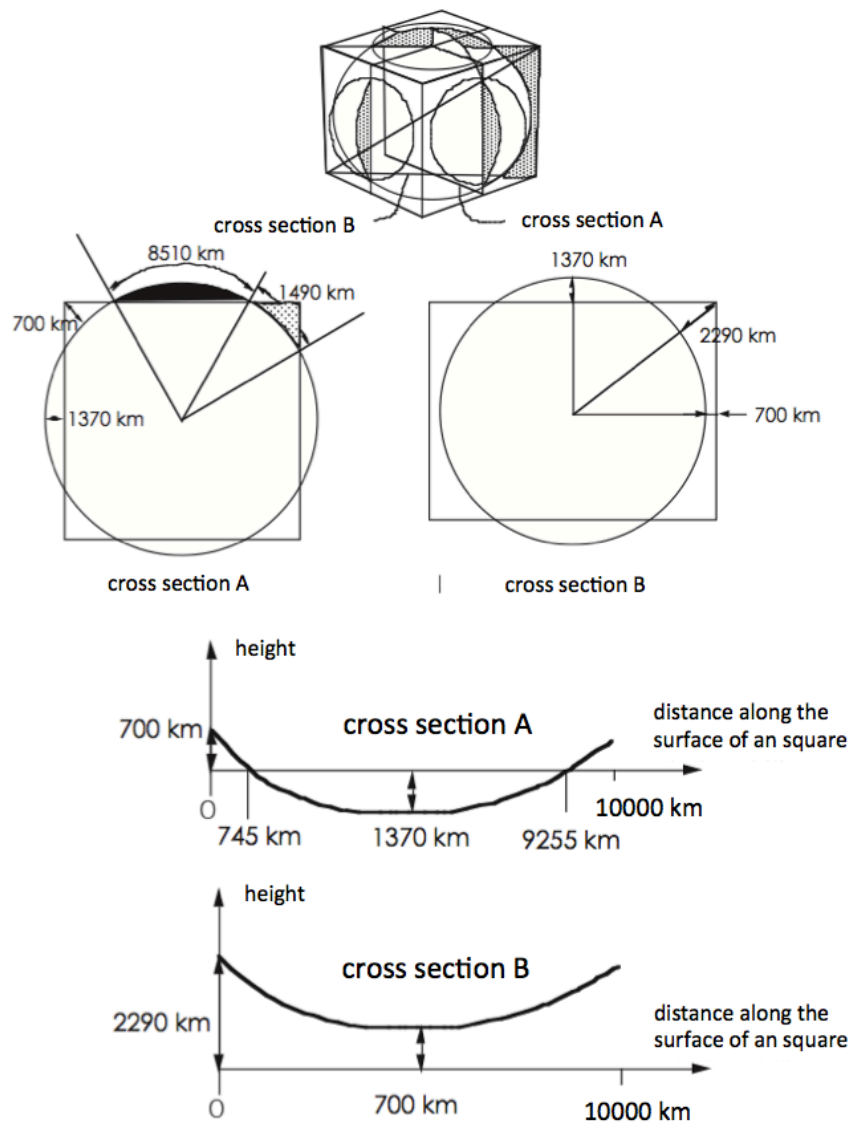

Figure 7: vertical distribution relative to the equal gravitational potential surface. (Assuming a geoid of the Earth at a equal gravitational potential surface that is represented by the spherical surface in the top diagram.)

4. The atmosphere and the ocean that cover the cubic Earth

4.1 Assumptions

The atmosphere and the ocean cover the surface of the Earth. The atmosphere covers the entire Earth. The ocean covers about 70% of the Earth's surface. This is why the Earth is called the water planet.

If the cubic Earth has the same amount of the atmosphere and the ocean with the real Earth, how do they cover the surface? We consider the existence form of the atmosphere and the ocean on the cubic Earth under the following assumptions.

Assumption1: The total amount of sea water is present on one side.
Assumption2: The total amount of air is equally distributed on all six sides.

The total amount of air $= 5.1 \times 10^{18}$ kg
The total amount of seawater $= 1.35 \times 10^{9}$ km^3

4.2 Form of the ocean

The seawater can be treated as incompressible fluid approximately. So, we assume that the same volume of the seawater as the real Earth is present in the cubic Earth. The form of sea level is a geoid. Because it is approximated by a spherical geoid, seawater piles up in the form of a convex lens on the center of the square (Figure 8).
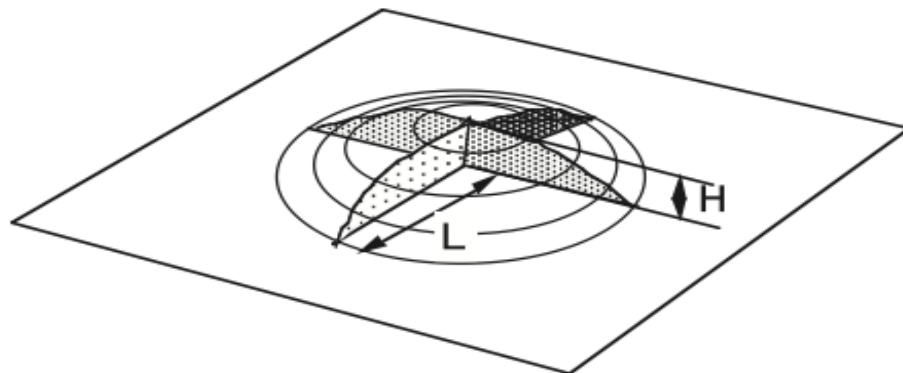

Figure 8: Seawater piled up in the form of a convex lens on the center of the square

Let L the radius of the convex lens, H the thickness of the convex lens, then the volume V is given approximately by

$$
\begin{aligned}
V &= \int_{R-H}^{R} \pi\left(R^2 - x^2\right) dx \\
&= \pi R^2 H - \frac{\pi}{3}\left[x^3\right]_{R-H}^{R} \\
&= \pi R^2 H - \frac{\pi}{3}\left(R^3 - \left(R^3 - 3R^2 H + 3RH^2 + H^3\right)\right) \qquad (4) \\
&= \pi\left(RH^2 - \frac{1}{3}H^3\right) \\
V &\approx \pi R H^2
\end{aligned}
$$

The coordinates are shown in Figure 12. R is the distance (approximated as part of the spherical surface) from the center of the Earth to the surface of the sea.

Firstly, let us consider the thickness H. The volume of the ocean $1.35 \times 10^9 \mathrm{km}^3$ is substituted into equation (4), and $R = 5{,}000$ km as a first approximation of $R = 5{,}000$ km $+ H$, then

$$
1.35 \times 10^9 \mathrm{km}^3 = 3.14 \times 5000\mathrm{km} \times H^2
$$
$$
\rightarrow H^2 = \frac{1.35 \times 10^9 \mathrm{km}^3}{3.14 \times 5000\mathrm{km}} = 0.086 \times 10^6 \mathrm{km}^2
$$
$$
\rightarrow H = 0.29 \times 10^3 \mathrm{km} = 290\mathrm{km}
$$

Next, $R = 5{,}290$ km as the second approximation, then

$$
1.35 \times 10^9 \mathrm{km}^3 = 3.14 \times 5290\mathrm{km} \times H^2
$$
$$
\rightarrow H^2 = \frac{1.35 \times 10^9 \mathrm{km}^3}{3.14 \times 5290\mathrm{km}} = 0.081 \times 10^6 \mathrm{km}^2
$$
$$
\rightarrow H = 0.29 \times 10^3 \mathrm{km} = 285\mathrm{km}
$$

Secondly, let us consider the radius L.

$$
\begin{aligned}
R^2 &= L^2 + (R - H)^2 \\
L^2 &= 2RH - H^2
\end{aligned}
\qquad (5)
$$

is satisfied, then

$$
L^2 = 2 \times 5285\mathrm{km} \times 285\mathrm{km} \text{-} 285\mathrm{km} \times 285\mathrm{km}
$$
$$
\rightarrow L = \sqrt{2931225}\mathrm{km} = 1712\mathrm{km}
$$

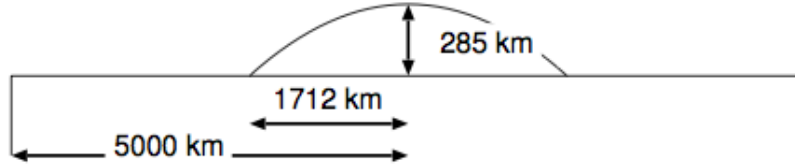is obtained. A cross-sectional view of the ocean is shown in Figure 9.

Figure 9:   A cross-sectional view of the ocean.

The area ratio of ocean to the side surface is

$$\pi \times 1712^2 / 10000^2 = 0.092$$

It is only to cover the 9.2% of one side surface.

4.3 Atmosphere on the side without the ocean

4.3.1 Relationship between the surface pressure and the total amount of air

Pressure is expressed as a height of mercury. Let V_m the total amount of the mercury, and R_E the radius of the Earth, then

$$
\begin{aligned}
V_m &= 0.76\text{m} \times 4\pi R_E^2 \\
&= 0.76\text{m} \times 4 \times 3.14 \times (6400000\text{m})^2 \\
&= 3.91 \times 10^{14}\text{m}^3
\end{aligned}
$$

Assuming that it is equally distributed on each side of the cubic Earth, the volume of the atmosphere on one side is

$$\frac{1}{6} V_m = 6.52 \times 10^{13}\text{m}^3 \tag{6}$$

Let us discuss how the atmosphere piles up on the side without the ocean. The atmosphere piles up in a convex lens as well as the ocean. We use a coordinate system with an origin at the center of a square. The z-axis is taken vertically upward and the x-axis is taken in the direction perpendicular to the direction of the vertical axis (z-axis) as shown in Figure 10. Note that the ground surface is different from the plane defined by z = 0 in this coordinate system.
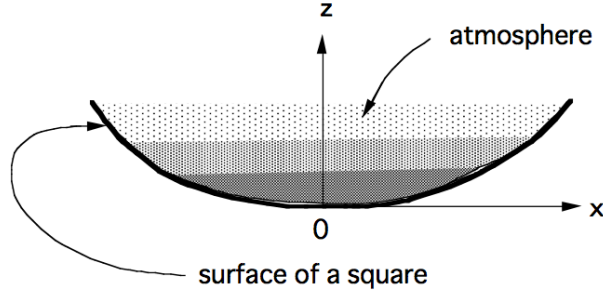
15

Figure 10: Coordinate system to investigate the atmospheric layer. The z-axis is taken in the direction of the gravity.

Vertical distribution of pressure along the z-axis is written as

$$h(z) = h_0 \exp\left(-\frac{z}{H_a}\right) \qquad (7)$$

Pressure is given as a height of mercury $h(z)$. $h_0 = h(0)$ in (7) corresponds to the surface pressure in the deepest layer. H_a is a constant (with dimension of length) called the scale height. H_a will be explained in the next section. The atmospheric pressure decreases exponentially with altitude. It is considered that by giving the total amount of the air, to determine the h_0. To do that, surface pressure is integrated along the square surface. If $x(z)$ is the altitude of the ground in Figure 10,

$$x^2 + (R - z)^2 = R^2 \qquad (8)$$

holds because of the spherical surface on x. R is the radius of the sphere. When $z \ll R$, we obtain

$$z = \frac{x^2}{2R} \qquad (9)$$

Spherical surface near the origin will be approximated to be a paraboloid. By substituting equation (7) to (9), the surface pressure is obtained.

$$h(x) = h_0 \exp\left(-\frac{x^2}{2RH_a}\right) \qquad (10)$$

The total amount of the air that piles up on the surface of a square is obtained by integrating the surface pressure. Paraboloid away from the origin becomes poor approximation, but the pressure also reduces with

16

distance from the origin, so that the contribution of the integral is small. Therefore, it is expected that the paraboloid gives a good approximation. With the same reason, integration range can be extended to infinity. Thus,

$$\frac{1}{6}V_m = \int_0^\infty h(x)2\pi x\,dx \tag{11}$$

$$= 2\pi h_0 \int_0^\infty x\exp\left(-\frac{x^2}{2RH_a}\right)dx$$

$$= 2\pi h_0 \left[-RH_a\exp\left(-\frac{x^2}{2RH_a}\right)\right]_0^\infty$$

$$= 2\pi h_0 RH_a$$

h_0 (surface pressure at the center of the face) depends on the H_a.

4.3.2 Scale height

Scale height H_a represents the thickness of the atmosphere. It is obtained from the equation of hydrostatic equilibrium:

$$\frac{dp}{dz} = -\rho g \tag{12}$$

and the equation of state of gas:

$$p = \rho R_g T \tag{13}$$

where ρ is the density of air at altitude z. And R_g is the gas constant, T is temperatures (with unit K). If T is constant, the result is

$$p(z) = p_0 \exp\left(-\frac{g}{R_g T}z\right) \tag{14}$$

The scale height is given by

$$H_a = \frac{R_g T}{g} \tag{15}.$$

By substituting $R_g = 287$ m/(K s²), $T = 300$ K, and $g = 9.8$ m/s²

$$H_a = \frac{287\,\text{m}^2\text{s}^{-2}\text{K}^{-1} \times 300\,\text{K}}{9.8\,\text{ms}^{-2}} = 8786\,\text{m} \tag{16}$$

Note that the scale height is the altitude that the pressure becomes 1 / e of the surface pressure. $T = 300$K is tentative, because the temperature distribution of the cubic Earth is unknown at this stage.

17

4.3.3 Surface pressure and pressure distribution

It is possible to estimate the surface pressure (h_0), if the scale height H_a is given. The result is

$$h_0 = \frac{V_m}{12\pi R H_a}$$
$$= \frac{3.91 \times 10^{14}\,\mathrm{m}^3}{12\pi \times 5.285 \times 10^6\,\mathrm{m} \times 8786\,\mathrm{m}} = 223\,\mathrm{m} \tag{17}$$

It means that the surface pressure in the deepest part of the atmosphere is equivalent to 223 m of the mercury, i.e., 294 atm. The vertical distribution of pressure is

$$p(z) = 294 \exp\left(-\frac{z}{8786\,\mathrm{m}}\right)\mathrm{atm}$$
$$\rightarrow \ln p(z) = \ln 294\,\mathrm{atm} - \frac{z}{8786\,\mathrm{m}} \tag{18}$$
$$\rightarrow z = 8786 \ln \frac{294\,\mathrm{atm}}{p(z)}\,\mathrm{m}$$

Altitude at 1atm   z = 49.9 km
Altitude at 0.5atm z = 56.0 km
Karman line   z = 171 km

Karman line is the altitude that the atmospheric pressure is 10^{-6} atm. It is regarded as the boundary between the atmosphere and the space.

The horizontal distribution of the pressure is obtained from the equation (9).

Radius at 1atm $x = \sqrt{2Rz} = \sqrt{2 \times 5.285 \times 10^3 \times 49.4}\,\mathrm{km} = 726\,\mathrm{km}$
Radius at 0.5atm $x = \sqrt{2Rz} = \sqrt{2 \times 5.285 \times 10^3 \times 56.0}\,\mathrm{km} = 769\,\mathrm{km}$
Radius of the Karman line $x = \sqrt{2Rz} = \sqrt{2 \times 5.285 \times 10^3 \times 171}\,\mathrm{km} = 1340\,\mathrm{km}$

In summary, the altitudes where the pressures are 1 atm and 0.5 atm are 49.9 km and 56.0 km, respectively. The radii that surface pressures become 1 atm and 0.5 atm are 769 km and 726 km from the center of the square,

respectively as shown in Figure 11.

The atmosphere on the cubic Earth is quite different from that on the real Earth. In the real Earth, the surface pressure is 1 atm anywhere at sea level. On the other hand, in the case of the cubic Earth, the pressure is 294 atm at the center of the square. The pressure decreases with distance from the center, being 1 atm at about 730 km away from the center. The more distant from there, the pressure is further reduced. It is 0.5 atm at about 770 km, and away from it, the air becomes thinner and thinner. Since the gas expands without limit, some air molecules are present at the edge of a square, but it should be regarded as vacuum. We can live in the atmospheric pressure between 0.5 atm and 1 atm, we can live in the circular belt of about 40 km width away 730 km from the center on the cubic Earth without the ocean. The area is called the habitable zone.
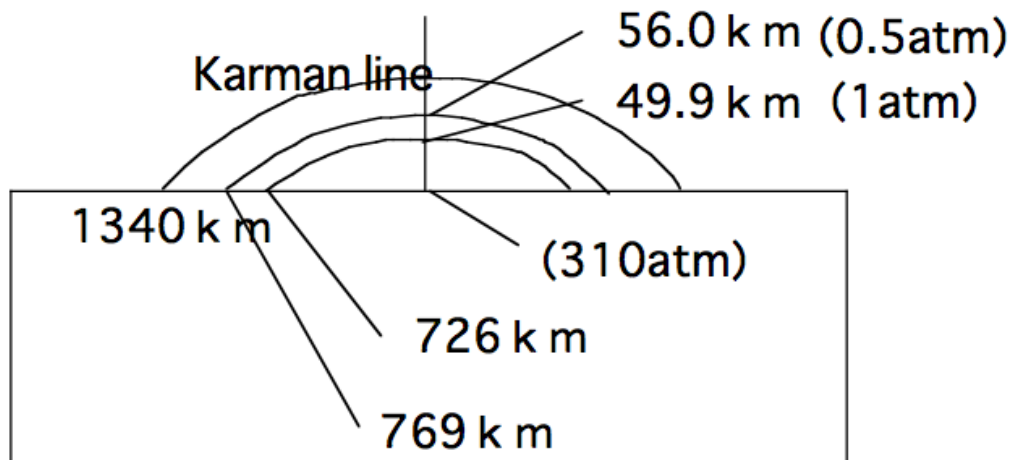

Figure 11: Cross-sectional view of the atmosphere

4.4 Atmosphere on the side with the ocean

In the center of the side where there is the ocean, the ocean is in the form of convex lens. The atmosphere piled on. The atmospheric pressure at sea level is the same everywhere. The mass of air covered over the land is very small compared to the mass of the atmosphere over the oceans. Therefore, the mass of the atmosphere is approximated by the mass of the air over the ocean.

The surface area of the sea is given by

$$surface\ area\ of\ convex\ lens = \int_0^\alpha 2\pi R^2 \sin\theta\, d\theta = -2\pi R^2 [\cos\theta]_0^\alpha = 2\pi R^2 (1-\cos\alpha) \quad (19)$$

with the symbols shown in Figure 12. Since R = 5,000 km + 285 km = 5,285 km and $\cos\alpha$ = 5,000/5,285 = 0.9461,

$$area\ of\ convex\ lens = 2\pi \times (5285\text{km})^2 (1-0.9461) = 9.46 \times 10^6\, \text{km}^3 \quad (20)$$

The height of mercury column that piles up on it is

$$\begin{aligned} h &= (volume\ of\ mercury) \div (surface\ area) \\ &= 6.52 \times 10^{13}\, \text{m}^3 \div 9.46 \times 10^{12}\, \text{m}^2 \\ &= 6.89\, \text{m} \end{aligned} \quad (21)$$

The surface pressure is

$$6.89\,\text{m} \div (0.76\,\text{m/atm}) = 9.07\text{atm} \quad (22)$$

By substituting 9.07atm to p_0 in equation (14), the vertical distribution of atmospheric pressure is obtained as

$$\begin{aligned} p(z) &= 9.07 \exp\left(-\frac{z}{8786\text{m}}\right)\text{atm} \\ &\rightarrow \ln p(z) = \ln 9.07\text{atm} - \frac{z}{8786\text{m}} \\ &\rightarrow z = 8786 \ln\frac{9.07\text{atm}}{p(z)}\,\text{m} \end{aligned} \quad (23)$$

Altitude at 1 atm $\qquad\qquad z = 8786 \times \ln(9.07/1)\text{m} = 19.4\text{km}$

Altitude at 0.5 atm $\qquad\qquad z = 8786 \times \ln(9.07/0.5)\text{m} = 25.5\text{km}$

Altitude of the Karman line $\quad z = 8786 \times \ln(9.07/10^{-6})\text{m} = 141\text{km}$
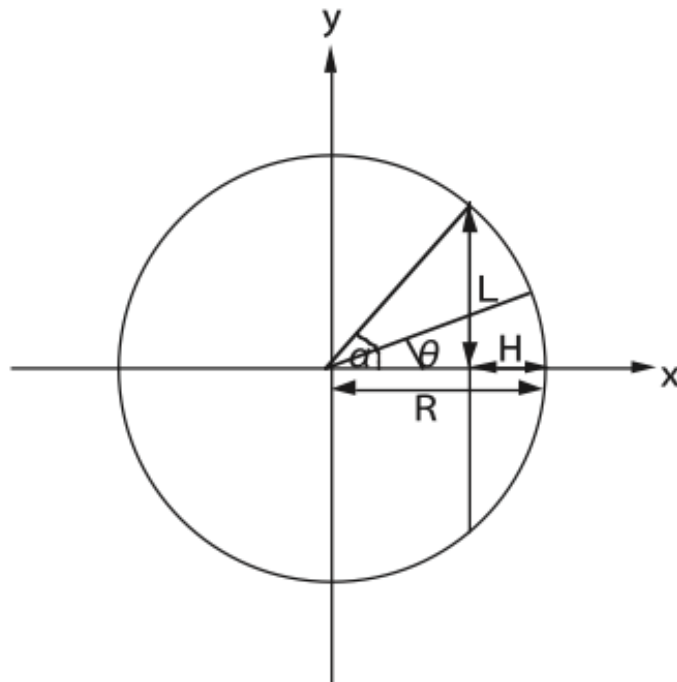
Figure 12: The definitions of the symbols

The surface pressure depends on the distance from the coast line.

Distance for 1 atm

$$\rightarrow x = \sqrt{(5285+19.4)^2 - 5000^2}\,\text{km} - 1712\text{km} = 59.1\text{km}$$

Distance for 0.5 atm

$$\rightarrow x = \sqrt{(5285+25.5)^2 - 5000^2}\,\text{km} - 1712\text{km} = 77.2\text{km}$$

Distance of Karman line

$$\rightarrow x = \sqrt{(5285+141)^2 - 5000^2}\,\text{km} - 1712\text{km} = 395\text{km}$$

Figure 14 shows the vertical cross-section of the atmosphere and the ocean. The habitable zone is a circular belt with a 20 km width located distance of approximately 60 km distant from the coastline.
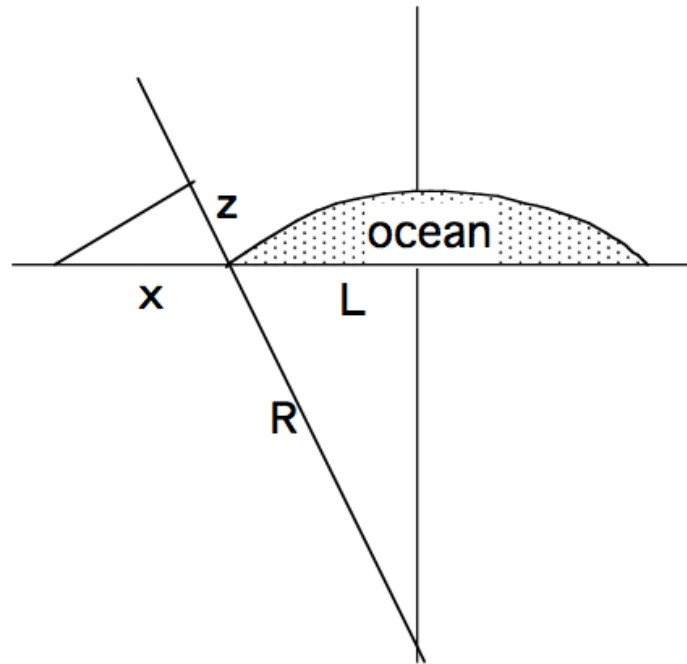
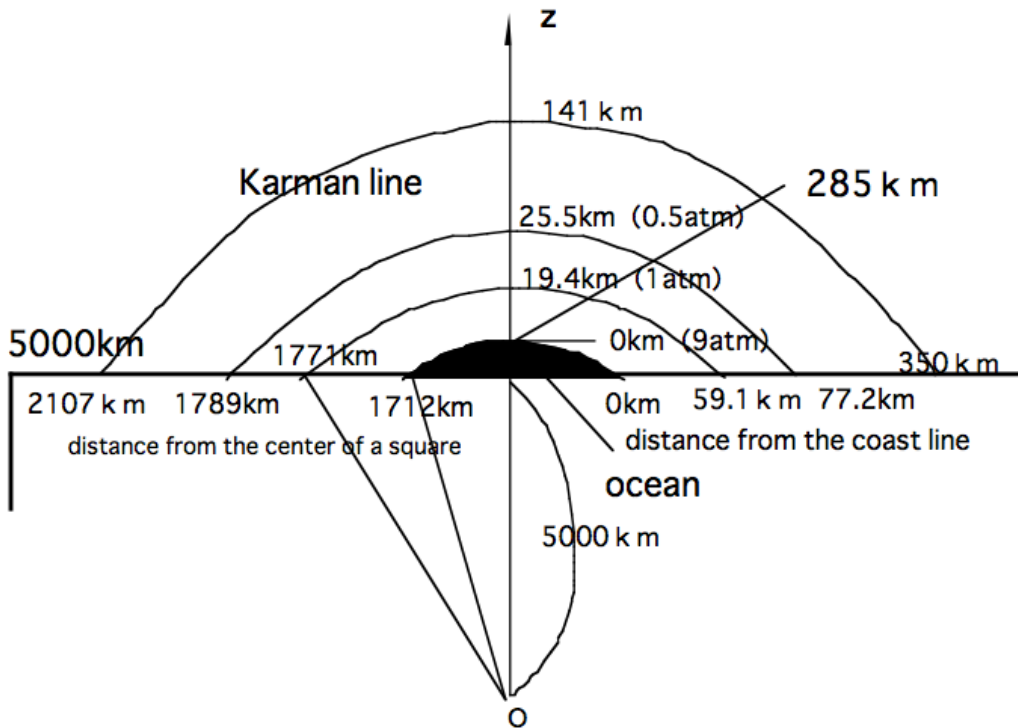Figure 13: Relationship of horizontal distance and altitude

Figure 14: Cross-section of the atmosphere in certain aspects of the sea
Insolation

22

5. Insolation

5.1 Angle of sunlight

  The cubic Earth revolves at an angle 23.4 degrees to the ecliptic plane as shown in Figure 15. Therefore, the angle of sunlight varies with the seasons. If the direction of the rotation axis is fixed, the direction in which the coming of the sun's rays changes depending on the season as shown in Figure 16.
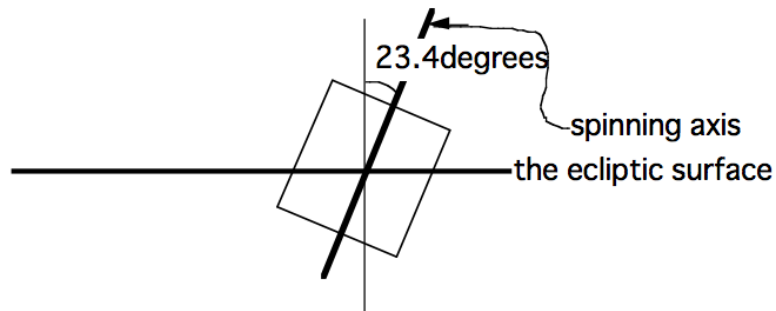

Figure 15: The cubic Earth revolves at an angle 23.4 degrees to the ecliptic plane. Seasonal change occurs for this.
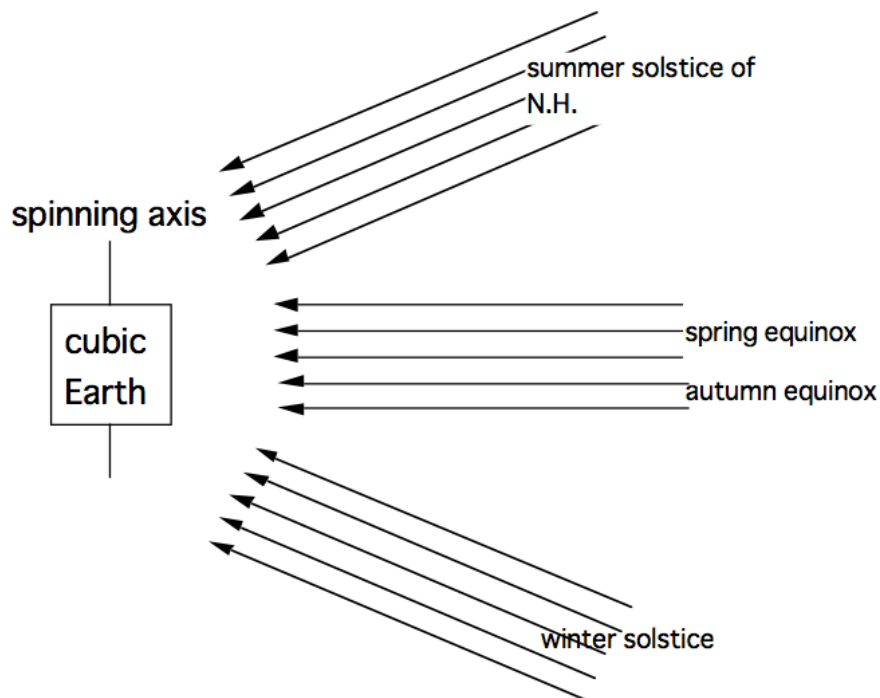

Figure 16: If the axis of rotation is fixed, the direction of the sun's rays is changed depending on the season.

5.2 Amount of the solar energy given on the side plane

Energy flux of the solar radiation given to the Earth is expressed as [(the projected area of the Earth)×(solar constant)]. The solar constant $S_0 = 1,370$ W/m² is energy flux of the solar radiation through unit area [1 m²] at the outer boundary of the atmosphere. The projected area of the Earth is the area of the Earth's shadow projected on a screen placed behind of the Earth perpendicular to the sun's rays. If the Earth is spherical, the projected area is given by πR_E^2, independent of the time and season, where R_E is the radius of the Earth.

In the case of the cubic Earth, the projected area varies depending on the time and season. In order to know its properties, it is necessary to determine the projected area. Before considering the whole cubic Earth, let us calculate the projected area of each surface of the cubic Earth.

To calculate the projected area of the plane without the poles, we consider only a square unit area is in the universe, rotating about the spin axis of the central portion of the side surface as shown in Figure 17.
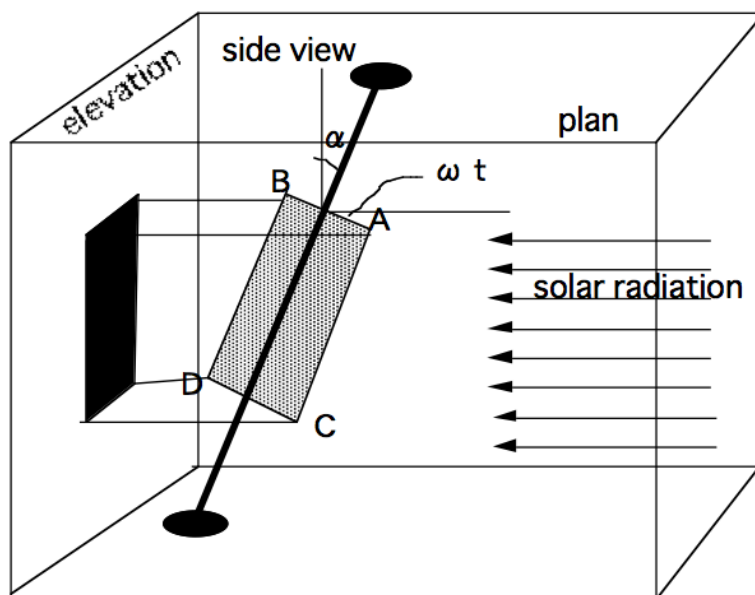

Figure 17: We consider a square ABCD which is a side of the cubic Earth rotating around an axis in the center of the square. The length of one side of the square is unit long. The rotation axis is inclined by an angle α to the sun's rays. The square makes an angle ωt for the direction of the sunlight. Figure 18 shows the plan and elevation of the configuration.

Let t the time elapsed from the time of sunrise, and ω the angular velocity of the rotation of the square. Figure 17 shows ωt which is the angle between the square and the sun's rays. Area of shaded elevation view of Figure 17 is the projected area of the square. This area D_0 is given in

$$D_0 = \cos\alpha \times \sin\omega t \qquad (24)$$

Since one side is in the square of 10,000 km, the projected area D is

$$D = \cos\alpha \times \sin\omega t \times 10^8 \, km^2 \qquad (25)$$

Flux of the solar radiation through unit area [1 m²] of this surface is

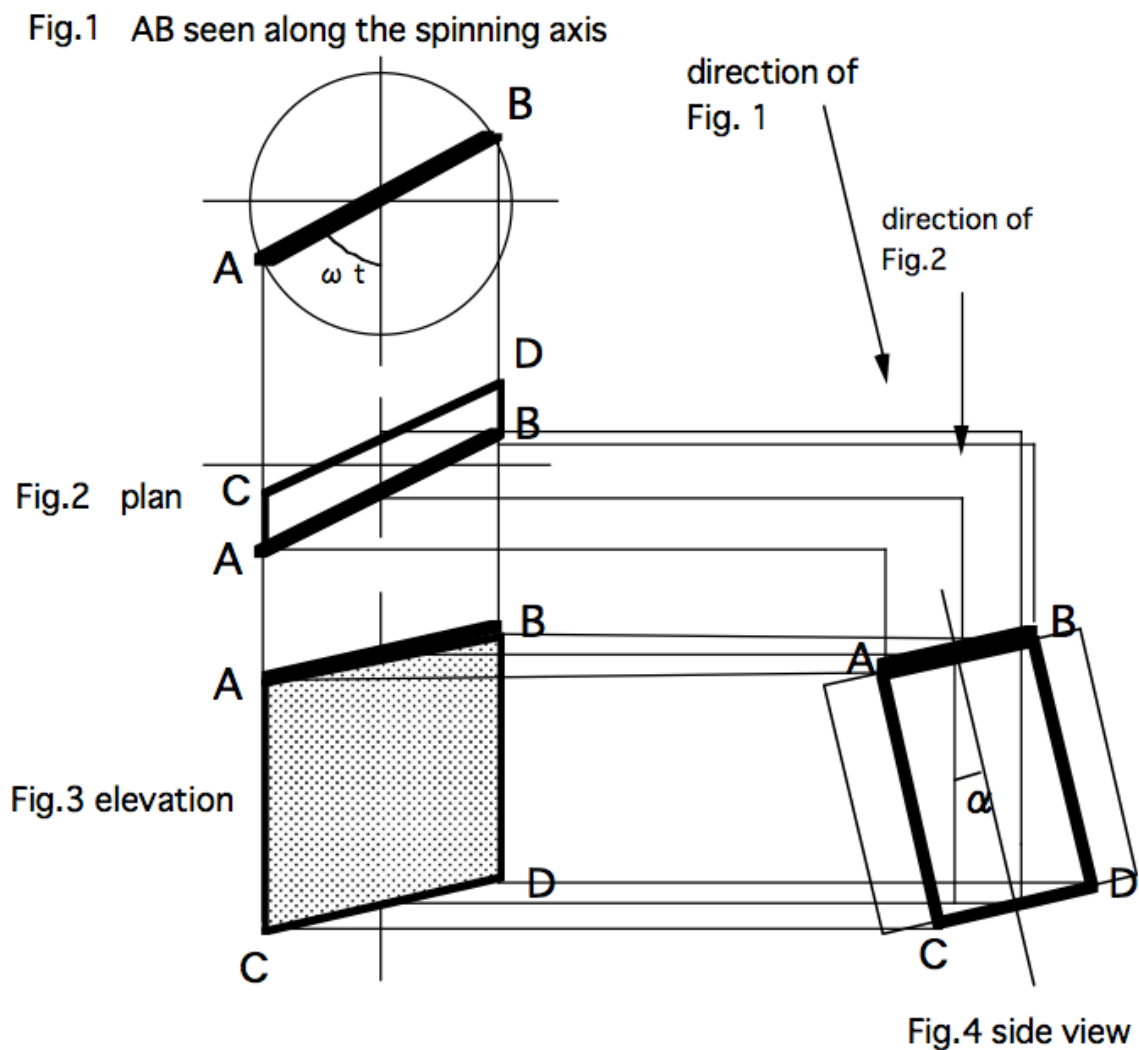$$S = S_0 \cos\alpha \times \sin\omega t \qquad (26)$$

Figure 18: Plan view, elevation view, side view that has been defined in Figure 17

5.3 Amount of the solar energy given on the polar plane

The polar plane (surface that has a pole) also rotates, so the projected shape varies. However, if α is fixed, the projected area does not depend on ωt. As shown in Figure 19 the projected area D of D_0 with an arbitrary shape is given by

$$D = D_0 \sin \alpha \qquad (27)$$

The projected area of the pole surface with the unit area is given by equation (27) and does not dependent on time.
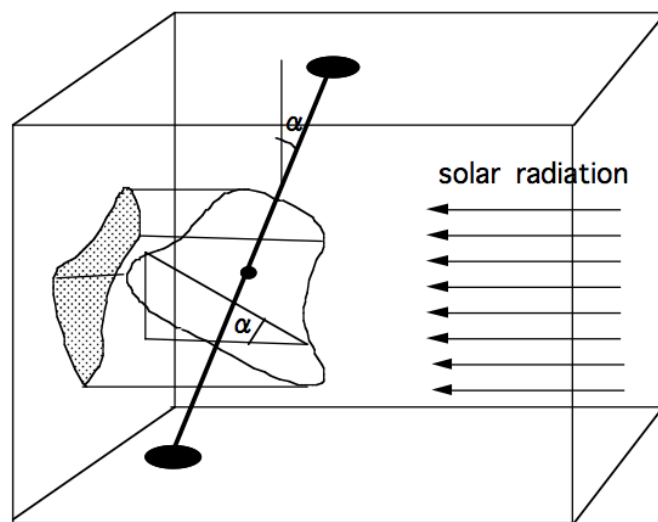

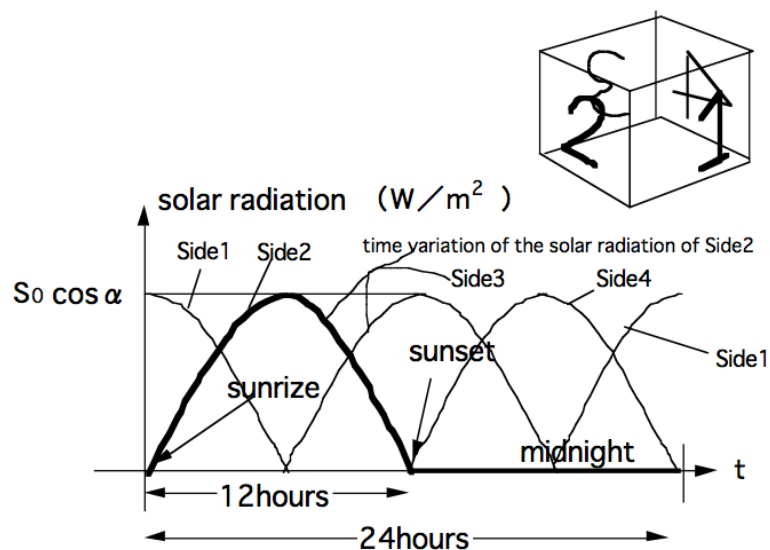Figure 19: Relationship between D_0 and D

Figure 20: Diurnal variations of the side plane. Graphs correspond to numbers on the side of the cube in the upper right insert.

5.4 Amount of the solar energy given on the cubic Earth

Sunlight is always shining on the three surfaces of the cubic Earth: side planes and a polar plane. Projected area D_{cube} is the sum of the projected area of the three planes as shown in Figure 20.

$$
\begin{aligned}
D_{cube} &= 10^8 \, \text{km}^2 \times (\sin\alpha + \cos\alpha \times \sin\omega t + \cos\alpha \times \sin(\omega t + \pi/2)) \\
&= 10^8 \, \text{km}^2 \times (\sin\alpha + \cos\alpha \times \sin\omega t + \cos\alpha \times \cos\omega t) \\
&= 10^8 \, \text{km}^2 \times (\sin\alpha + \cos\alpha(\sin\omega t + \cos\omega t)) \\
&= 10^8 \, \text{km}^2 \times (\sin\alpha + \sqrt{2}\cos\alpha \sin(\omega t + \pi/4))
\end{aligned}
\tag{28}
$$

The second term corresponds to the projected area of side planes. They vary as shown in Figure 21. Average area under the 24 hour period is given by

$$
\begin{aligned}
&\frac{4}{\pi}\int_0^{\frac{\pi}{4}} \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)d\theta \\
&= \frac{4}{\pi}\sqrt{2}\left[-\cos\left(\theta + \frac{\pi}{4}\right)\right]_0^{\frac{\pi}{4}} \\
&= \frac{4}{\pi}\sqrt{2}\left(0 + \frac{1}{\sqrt{2}}\right) \\
&= \frac{4}{\pi} = 1.273
\end{aligned}
\tag{29}
$$

So, projected area of daily mean is given by

$$
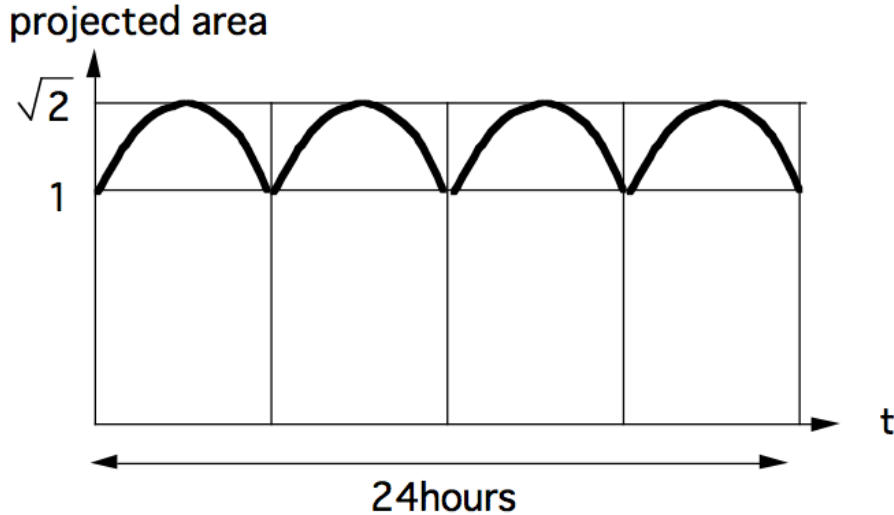D_{cube}(daily\ mean) = 10^8 \, \text{km}^2 \times (\sin\alpha + 1.273 \times \cos\alpha)
\tag{30}
$$

Figure 21: Time variation of the projected area of the side planes. It is normalized by 10^8 km² × cosα.

27

Let us compare the projected area in the winter solstice (= 23.4 degrees) and in the Spring Equinox (= 0 degree).

Projected area of the winter solstice = 10^8km^2 (sin 23.4 +1.273cos 23.4)

= 10^8 km^2 (0.40 + 1.273$\times$0.92) = 1.57$\times$$10^8$ km^2

Projected area of the Spring Equinox = 1.27$\times$$10^8$ km^2

The average of these two values gives approximately annual average of the projected area of the cubic Earth. That is, approximately,

Projected area of the annual average = 1.42$\times$$10^8$ km^2.

On the other hand, if the projected area of the Earth is a sphere with a radius R_E (= 6,400 km),

$$\pi R_E^2 = \pi \times 0.64^2 \times 10^8 \text{km}^2 = 1.29 \times 10^8 \text{km}^2,$$

the projected area is approximately equal to the projected area of the Spring Equinox of the cubic Earth. The annual average of the projected area of the cubic Earth is larger than the annual average of the spherical Earth.

5.5 Seasonal variation and diurnal variation of solar radiation

5.5.1 Seasonal changes of solar radiation in the polar plane

Atmosphere of the Earth is heated by solar radiation. The intensity shows seasonal variation, because the rotating axis is tilted. As shown in Figure 16, if the Earth's axis is fixed in the vertical direction, the direction of the sunlight changes with the seasons. Unlike spherical Earth, in the case of the cubic Earth, insolation characteristics of polar face and side surface are quite different. In the polar face, there is no alternation of day and night with the period of 24 hours. Six months in the daytime, and rest of the year is night. The year insolation changes as shown in Figure 22.
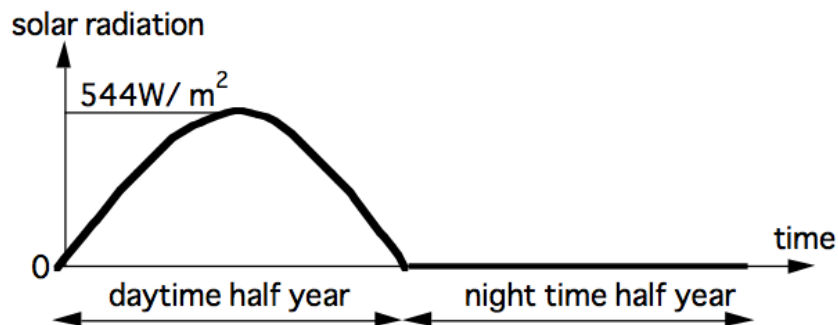

Figure 22: Seasonal changes in solar radiation in the polar plane

5.5.2 Seasonal changes of solar radiation in the side plane

Day and night are both 12 hours regardless of latitudes and seasons on the side surface of the cubic Earth. Diurnal change of solar radiation is shown in Figure 23. S_{max} (the maximum amount of solar radiation) depends on the season. The maximum value of S_{max} is 1370W/m² and is observed in Spring Equinox and the Autumn Equinox. The daily average is 436 W/m², which is smaller than the maximum value in the polar surface. The minimum value is 400 W/m² and is observed in winter solstice or summer solstice. The seasonal average is 418 W/m². The seasonal variation of the daily average amount of solar radiation is shown in Figure 24.
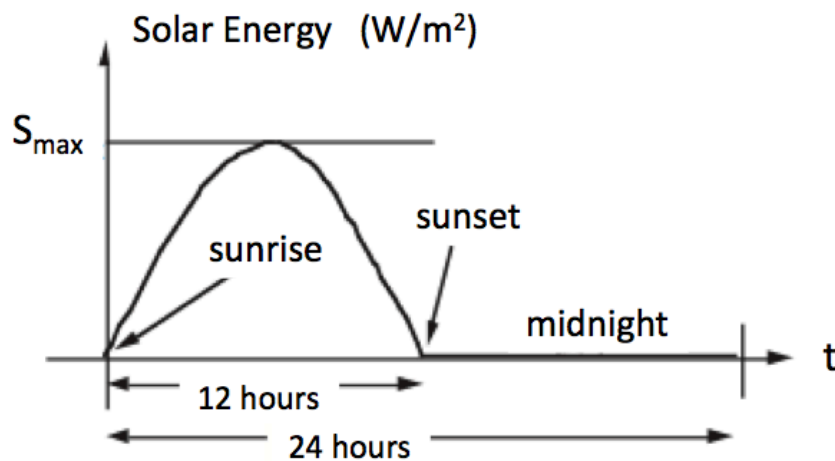
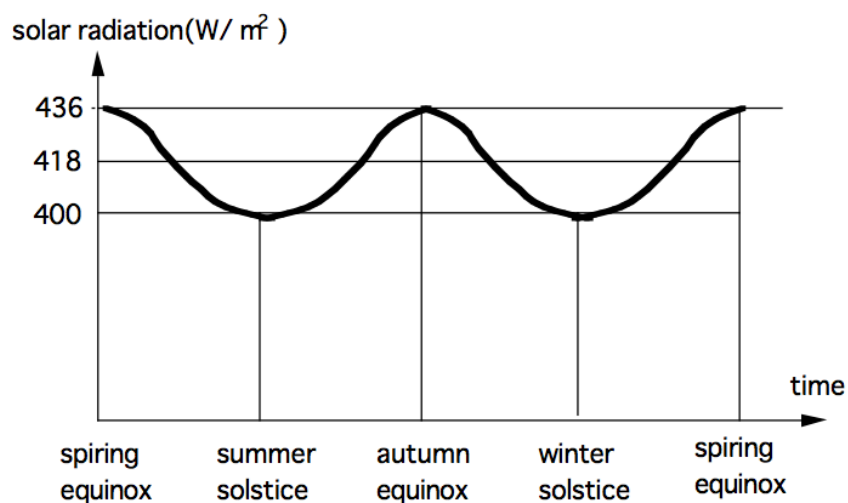Figure 23: Diurnal change of solar radiation in the side plane

Figure 24: Seasonal changes in the average daily amount of solar radiation in the side plane

29

6. The surface temperature of the cubic Earth

6.1 Radiative equilibrium

We feel warm in a sunny spot in a cold winter day. It is because the solar radiation has a power to heat objects. However, if the object continues to absorb solar radiation, the temperature rises without limit. In fact, the energy escapes in the form of infrared rays whose intensity is a function of temperature. This phenomenon is known as "thermal radiation". When thermal radiation energy escaping from the object becomes equal to the solar radiation energy, the temperature of the object falls into a steady state. This state is known as "radiative equilibrium state". The surface temperature of the cubic Earth is also determined by radiative equilibrium state.

In the real Earth, there is no region where land is in contact with the universe like the surface of the moon. On the other hand, the atmosphere covers only a part of the surface of the cubic Earth. In the first step, let us determine the temperature at the periphery of the square where there is no atmosphere and ocean.

6.2 Seasonal changes of the surface temperatures of the polar surface without the atmosphere

When the Earth's surface is heated by the solar radiation, heat penetrates into the interior of the Earth. The temperature change is described by the equation of heat conduction in the vertical direction.

The equation of the heat conduction can be written as

$$\rho c \frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2} \qquad (31)$$

with a z-axis as the vertical upward direction, where t is time, T is temperature, ρc is volume heat capacity of the rock, and κ is thermal conductivity of the rock. Heat balance of the Earth's surface can be written as

$$\kappa \frac{\partial^2 T}{\partial z^2} = S(1-A)f(t) - \sigma T_S^4 \qquad (32)$$

where T_s is surface temperature, S is solar constant, A is reflectivity of the

cubic Earth, and $f(t)$ is cosine of the angle of sunlight. σ is a constant of the Stefan-Boltzmann's law.

Seasonal variation of the surface temperature was calculated with constants:

$$S = 1370\,\text{W}/\text{m}^2$$
$$A = 0.3$$
$$\kappa = 3\,\text{W}/\text{K}/\text{m}$$
$$\rho c = 2.5 \times 10^6\,\text{J}/\text{K}/\text{m}^3$$
$$\sigma = 5.67 \times 10^{-8}\,\text{W}/\text{m}^2/\text{K}^4$$

The result is shown in Figure 25. The upper figure shows seasonal change of the temperature at the surface of the polar square. As expected from movement of the sun (see section 5.5), there is no diurnal variation. After the middle of the midnight sun (approximately 100 day from the vernal equinox), the temperature reaches to the maximum (275 K (2 ℃)), after that and reaches to the minimum (155 K (-118℃)) at the end of the polar night. We can live in the season with solar radiation, but in the polar night, the polar square is no longer a place where human can live.
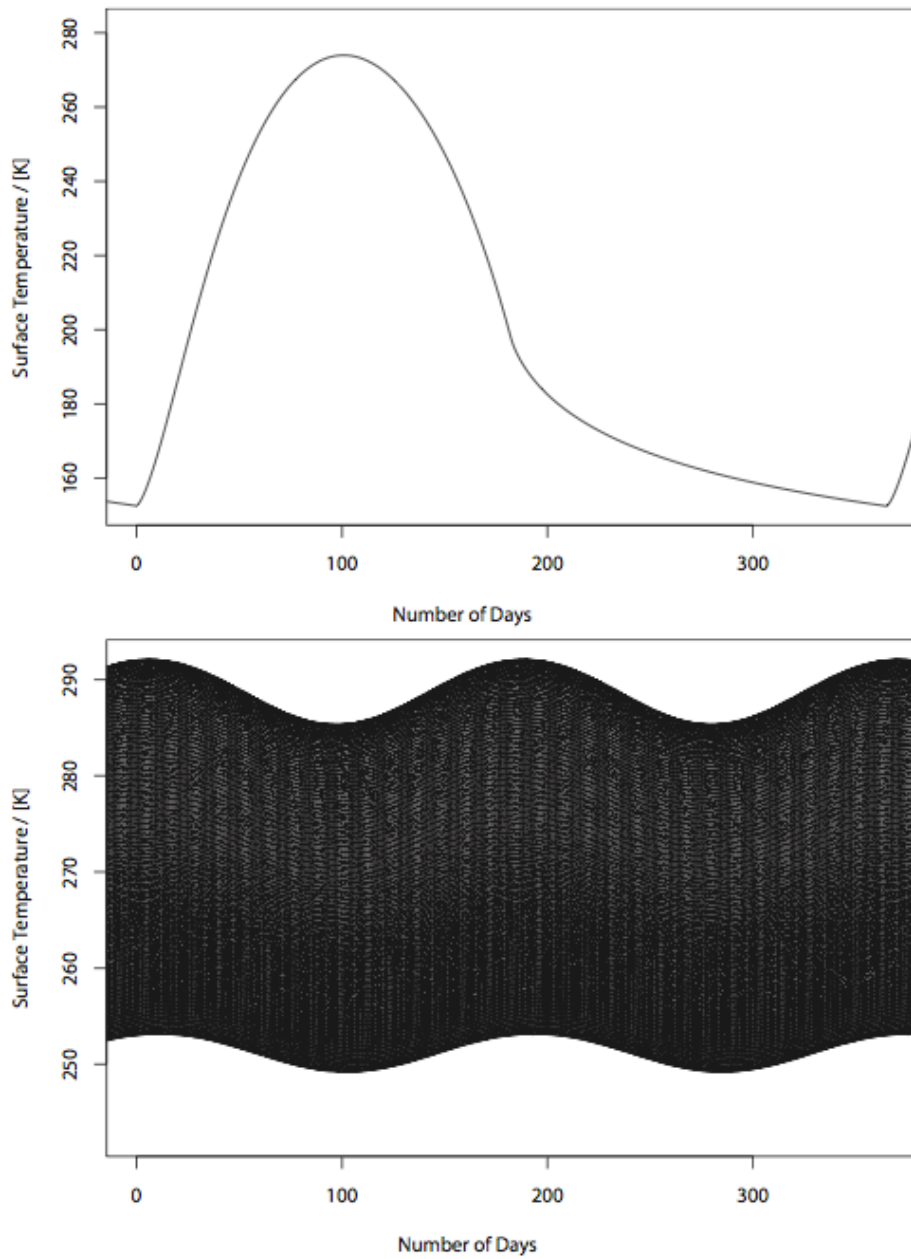
Figure 25: Time change of the surface temperature of the ground without the atmosphere. Above: the polar surface, below: side surface

6.3 Seasonal changes of the surface temperatures at the ground of the side square without the atmosphere

The seasonal change of the surface temperatures of the side square without the atmosphere is shown in the lower part of Figure 25. Unlike the real Earth, the seasons have a period of six months. The amplitude of the diurnal change is greater than the amplitude of the seasonal variation. It can be said that the side surface is in a temperature range in which we can live throughout the year, because its maximum temperature is about 290 K and the lowest temperature is about 253 K. The averaged temperature is slightly larger than radiative equilibrium temperature of the real Earth (255 K (-18 ℃)).

Temperature calculated here is the radiative equilibrium temperature of a surface without the atmosphere, the same result can be applied to a place with a thin air. The climate in the area between 800 km to 1,000 km away from the center of the center of the square will be close to the tropical climate of the real Earth.

6.4 Vertical distribution of temperature on the side square with an atmosphere without the ocean

The temperature is considered to be substantially constant along a geopotential surface. In other words, the isothermal surface covers the cubic Earth in a dome shape. As a result, the pressure and temperature also decrease gradually toward the periphery from the center of the square. This is like mountains in the real Earth where atmospheric pressure and temperature also decrease gradually.

Figure 26 shows the relationship between horizontal distance (distance from the center of the square) and altitude. A place at 1,000 km from the center of the cubic Earth corresponds to an altitude of 100 km above the center.

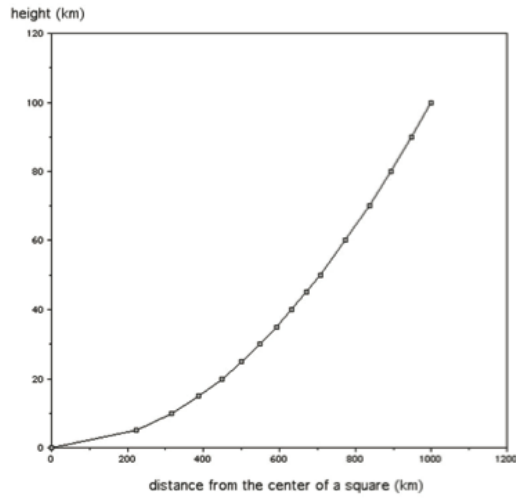height (km)

distance from the center of a square (km)

Figure 26: relationship between horizontal distance and altitude

Figure 27 shows a vertical distribution of the pressure at the center of the square when the scale height is assumed to be 8,786 m [*]. Figure 28 shows relationship of horizontal distance and surface pressure. It indicates that the atmospheric pressure is reduced when going outward from the center of the square. Figure29 shows an enlarged view of the same graph where atmospheric pressure is close to 1 atm at 700 km away from the center.

[*]: In the following, the surface pressure is calculated at 310 atm that was originally supposed instead of 294 atm. However, there is no qualitative difference in the results.
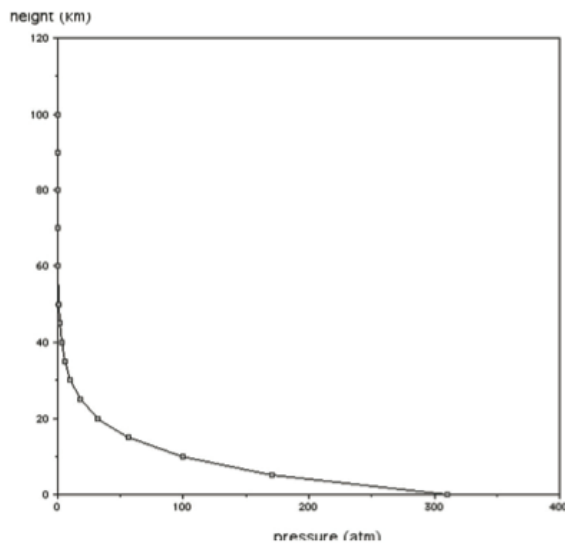

height (km)

pressure (atm)

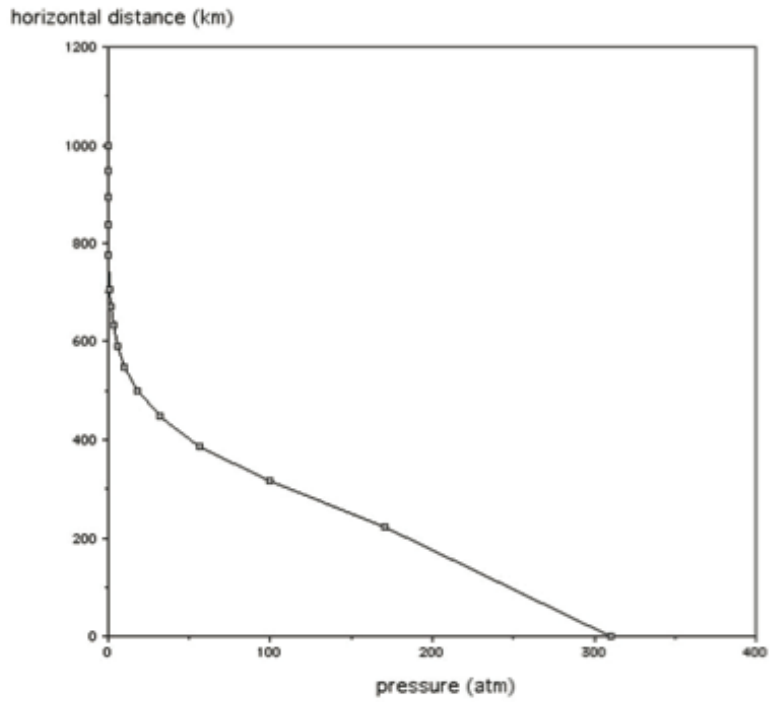Figure 27: Vertical distribution of atmospheric pressure

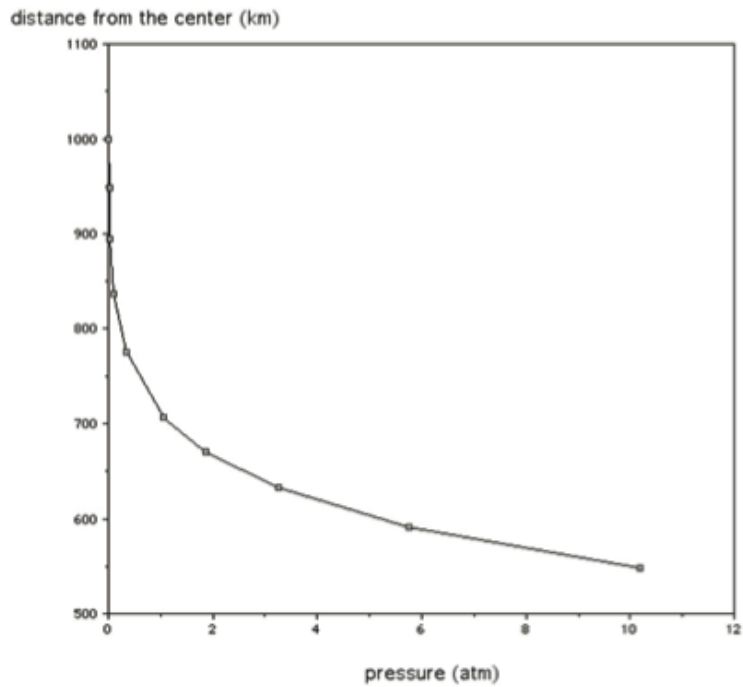Figure 28: Relationship between the horizontal distance and the surface pressure

Figure 29: Relationship between the horizontal distance and the surface pressure (Enlarged view of Figure 28 around 1 atmosphere)

Atmospheric temperatures depend on the amount of water vapor contained in the atmosphere. Water vapor is the most important greenhouse gas in the present Earth. Water vapor content determines the extent of the greenhouse effect. We calculated the temperature distribution subject to the effect of water vapor under following assumptions:

1. There is no wavelength dependency of the greenhouse gases for emitting thermal radiations.

2. Sunlight is absorbed only at the surface of the cubic Earth.

3. The emission rays can be approximated vertical directions (two-stream approximation). In other words, non-uniformity and the temperature difference in the horizontal direction is ignored.

4. Absorption coefficient of the radiation is constant. The pressure-dependence of the absorption coefficients is ignored.

5. Ratio of water vapor to the dry air is constant, assuming there is no source of water vapor such as the sea. In addition, it is assumed that the amount of water vapor in 1 atm is the same with that of the atmosphere of the real Earth. For example, amount of water vapor in the atmosphere of 100 atm is 100 times greater than that in the atmosphere of the real Earth.

6. Albedo is assumed to be This is due to the fact that the atmosphere is covered by clouds, as described below. Radiative equilibrium temperature is 201 K.

The result is shown in Figure 30. Surface temperature at the center of the square is 1,210 K. Temperature decreases almost linearly with height. This decrease rate is close to the dry adiabatic lapse rate. In the lower layer, the temperature lapse rate is 8.3 K / km.

Figure 31 shows the distribution of surface temperature along the equal gravity potential surface. Isotherms at the land surface distribute in the forms of concentric circles.
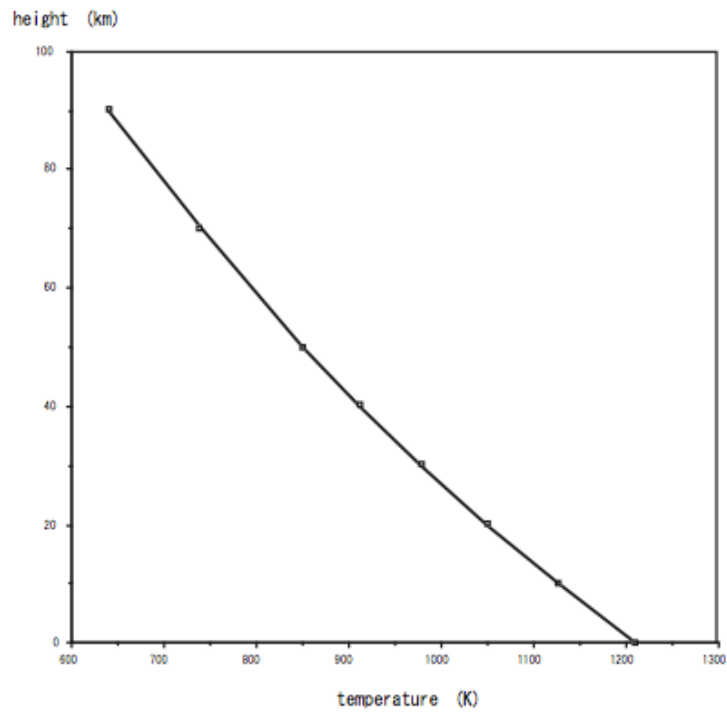
height (km)

Figure 30: Vertical distribution of temperature
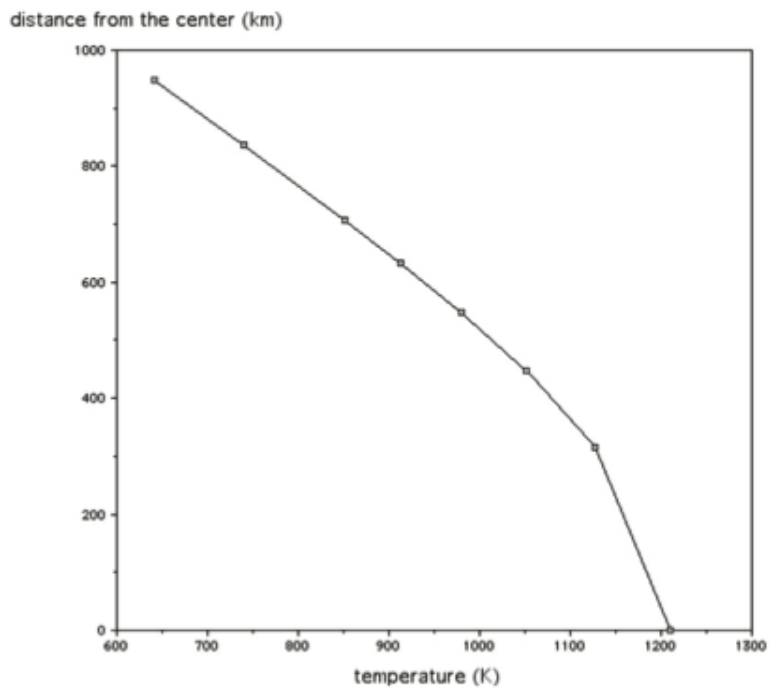
distance from the center (km)

Figure 31: Relationship with the distance from the center of the square and the ground temperature

6.5 Formation of clouds

Clouds play a important role in the Earth environment. They are;

1. Cloud amount controls albedo.
2. Clouds give influences to temperature through the process of greenhouse effect.
3. Clouds heat upper atmosphere by releasing latent heat.
4. Clouds produce precipitation on lands and sustain life on lands.

How do these effects work on the cubic Earth? Understanding these processes will help us to understand atmospheric phenomena on the real Earth.

Can we see clouds in the sky of the cubic Earth? Figure 32 compares the water vapor pressure of the air on the cubic Earth with saturated water vapor pressure as a function of altitude. The water vapor decreases with altitude. The saturated water vapor pressure also decreases with altitude due to the temperature decrease. The water vapor is saturated at the height of 215 km. There is a cloud bottom at this altitude. The temperature at this altitude is about 300 K. Clouds cover in the form of a dome in the central region of a square. Clouds touch the ground at a place 1,100 km apart from the center of the square. Probably, the ground outside of this area is covered with fogs.
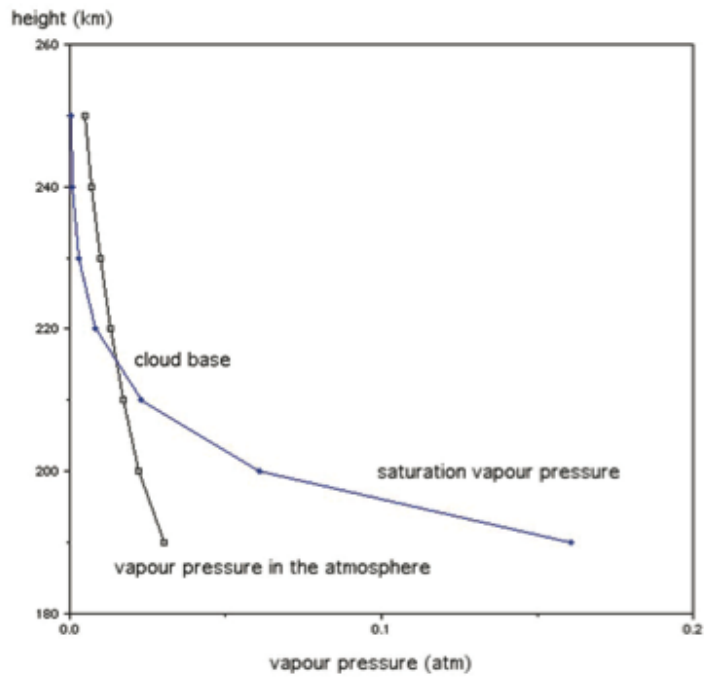
Figure 32: Vertical distribution of the amount of water vapor in the atmosphere (indicated by water vapor pressure) and saturated water vapor pressure.
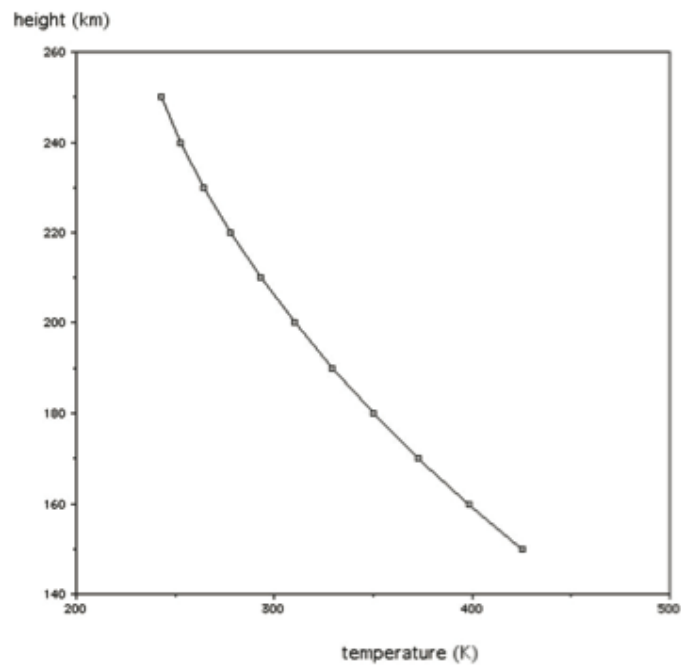

Figure 33: Vertical distribution of temperature in the altitude range corresponding to Figure 32.